亞洲大學 博士班資格考 試題

文件編號：AC-0054
機密等級：2 限閱

考試日期：112 年 09 月 06 日

考試科目	演算法	系所別	人工智慧博士學位學程	命題教師	

1. (30%) Give the best, worst, and average numbers of exchanges needed in Bubble Sort, which compares two adjacent elements and swaps them until they are in the intended order. The best-case and worst-case analyses are trivial. The average-case analysis can be done by the following process:

   (a) Define the inversion of a permutation. Let $a_1, a_2, \ldots, a_n$ be a permutation of the set $\{1, 2, \ldots, n\}$. If $i < j$, and $a_i > a_j$, then (a_i, a_j) is called an inversion of this permutation.

   (b) Point out the relationship between the probability that a given permutation has exactly k inversions and the number of permutations of n elements having exactly k inversions.

   (c) Derive the average number of exchanges needed for Bubble Sort. Prove your answer.

2. (30%) Consider two strings, A and B. The longest common subsequence (LCS) of two strings is the longest sequence of characters that appears in both strings in the same relative order, but not necessarily consecutively.

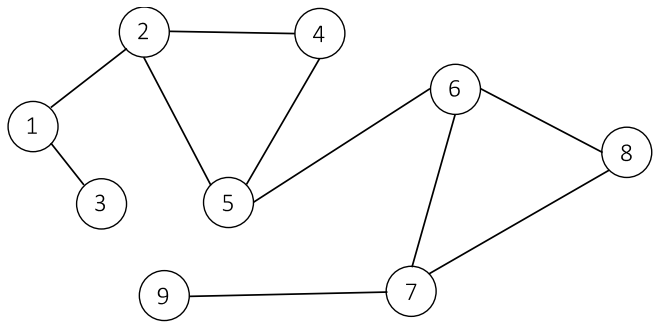
   (a) Write a function to find the length of the LCS between string A and string B.

   (b) What is LCS between A and B ("ABCDAB" and "BACDB")?

3. (20%) Given a connected graph G with n vertices and m edges. We say an edge of G is a bridge if the graph becomes a disconnected graph after removing the edge. For example, in the following graph, edges (3,1), (1,2), (5,6), (7,9) are bridges.

   (a) Give an $O(m^2)$ time algorithm that finds all the bridges

   (b) Give an O(m + n) time algorithm that finds all the bridges.



4. (20%) One day, highly intelligent alien came to earth and proposed an $O(n^{10})$ algorithm to solve the 3-SAT problem. Provide a short justification for the following claims.

   (a) P vs NP problem have been solved.

   (b) All NP problems are solvable in $O(n^{10})$ time.